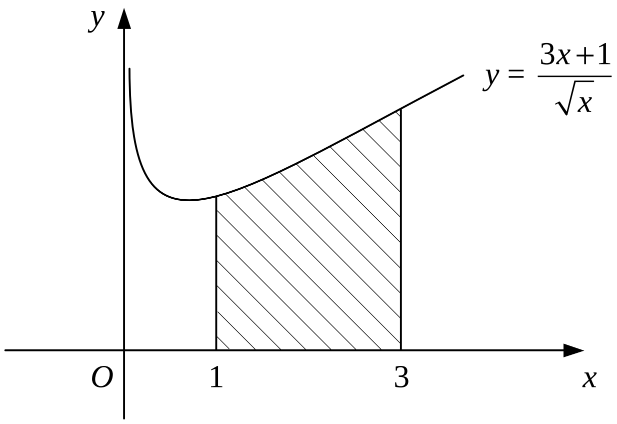
Core Mathematics 3 Paper A

1. Evaluate

$$\int_{2}^{15} \frac{1}{\sqrt[3]{2x-3}}\ \mathrm{d}x.$$ [5]

2.



The diagram shows the curve with equation $y = \frac{3x+1}{\sqrt{x}}$, $x > 0$.

The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$.

Find the volume of the solid formed when the shaded region is rotated through four right angles about the x-axis, giving your answer in the form $\pi(a + \ln b)$, where a and b are integers. [6]

3. A curve has the equation $y = (3x - 5)^3$.

(i) Find an equation for the tangent to the curve at the point P (2, 1). [4]

The tangent to the curve at the point Q is parallel to the tangent at P.

(ii) Find the coordinates of Q. [3]

4. Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} - x + 2 = 0$$

$$\ln (x + 3) - 2y - 1 = 0$$ [7]

5. *(i)* Find the exact value of x such that

$$3\tan^{-1}(x-2)+\pi=0.$$ [3]

(ii) Solve, for $-\pi<\theta<\pi$, the equation

$$\cos 2\theta-\sin\theta-1=0,$$

giving your answers in terms of π. [5]

6. The functions f and g are defined by

$$\mathrm{f}: x \to 3x-4, \quad x\in\mathbb{R},$$

$$\mathrm{g}: x \to \frac{2}{x+3}, \quad x\in\mathbb{R}, \quad x\neq -3.$$

(i) Evaluate fg(1). [2]

(ii) Solve the equation $\mathrm{gf}(x)=6$. [4]

(iii) Find an expression for $\mathrm{g}^{-1}(x)$. [2]

7. *(i)* Express $2\sin x° - 3\cos x°$ in the form $R\sin(x-\alpha)°$ where $R>0$ and $0<\alpha<90$. [3]

(ii) Show that the equation

$$\operatorname{cosec} x° + 3\cot x° = 2$$

can be written in the form

$$2\sin x° - 3\cos x° = 1.$$ [1]

(iii) Solve the equation

$$\operatorname{cosec} x° + 3\cot x° = 2,$$

for x in the interval $0\le x\le 360$, giving your answers to 1 decimal place. [4]

Turn over

8. The functions f and g are defined for all real values of x by

$$f : x \to |x - 3a|,$$

$$g : x \to |2x + a|,$$

where a is a positive constant.

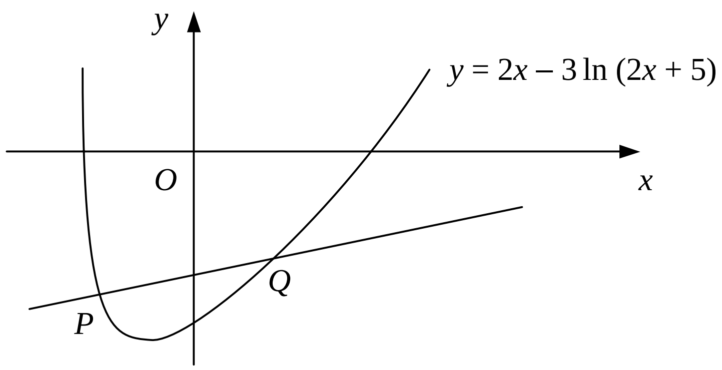
(i) Evaluate fg($-2a$). [2]

(ii) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, showing the coordinates of any points where each graph meets the coordinate axes. [4]

(iii) Solve the equation

$$\mathrm{f}(x) = \mathrm{g}(x).$$ [4]

9.



The diagram shows the curve with equation $y = 2x - 3\ln(2x + 5)$ and the normal to the curve at the point P $(-2, -4)$.

(i) Find an equation for the normal to the curve at P. [4]

The normal to the curve at P intersects the curve again at the point Q with x-coordinate q.

(ii) Show that $1 < q < 2$. [3]

(iii) Show that q is a solution of the equation

$$x = \tfrac{12}{7}\ln(2x + 5) - 2.$$ [2]

(iv) Use an iterative process based on the equation above with a starting value of 1.5 to find the value of q to 3 significant figures and justify the accuracy of your answer. [4]